E&M - Exam 4 [redacted]

Question 1 8/8

$P = 230W$

$V = 230V$

$R(T) = R_0(0.006T - 0.77)$

R at 300K

(1.) $T_{op} = 3000K$

$R_{op} = ?$ $\cancel{\left[= R_0(0.006 \cdot 3000 - 0.77) = 17.23 R_0 \right]}$

$P = VI = \frac{V^2}{R_{op}} \Rightarrow R_{op} = \frac{V^2}{P} = \frac{230^2}{230} = \underline{230\,\Omega}$

(2.) $R_0 = ?$

$R_0 = \frac{R}{0.006T - 0.77} \overset{\text{from 1.}}{=} \frac{230}{0.006 \cdot 3000 - 0.77} \doteq \underline{13.35\,\Omega}$

(3.) $P_{switch} = ?$

$R_0 = 13.35\,\Omega$

$P_{switch} = \frac{V^2}{R_0} = \frac{230^2}{13.35} \doteq \underline{3963\,W}$

(4.) The lamp is made for power of about 230W which is supplied to the lap when ~~the fillament~~ the lamp is in the operation and the fillament is hot - at that time (at temperature ~3000K), the resistance is high enough to keep the power around the right value at given supplied voltage.

However, when the voltage is applied to fillament in the lamp which is at room temperature (not hot yet - this happens when the lamp is switched on), the resistance is quite low, hence the power given to the lamp is very high. If the lamp isn't in the best state, this high power can easily cause the fillament to burn down. 1

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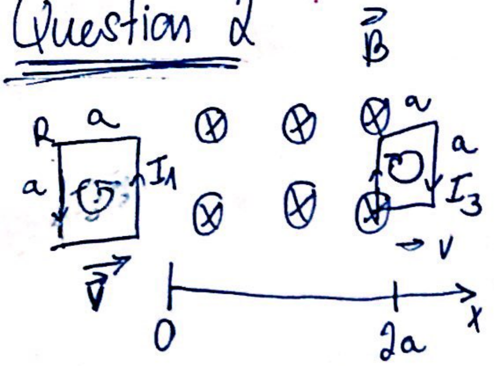
Question 2 9/10



$\mathcal{E} = ?$

$\vec{I}$ direction = ?

(1.) $\mathcal{E} = -\frac{d\phi}{dt}$

$\Phi = \int_S \vec{B} \cdot d\vec{a} = B \cdot a \cdot vt$

S ↳ area in magnetic field

B homogeneous

vt ↳ length of horizontal part in magnetic field (when loop moves at constant speed) after time t of movement, t = 0 when the front of the loop is at the border of the field

right part

$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d(Bavt)}{dt}$

magnitude: $\underline{\mathcal{E} = -Bav}$

~~Change in~~ $\vec{B}$ is increasing when entering the loop ⇒ change in $\vec{B}$ is in the direction of $\vec{B}$ (into page ⊗). ⇒ By Lenz's law, the induced current will be such that the induced $\vec{B}$ will oppose this change ⇒ direction of induced $\vec{B}$ is ⊙. By right hand rule, such field inside the loop is created by current in counter-clockwise ↺ direction. $\left(I_1 = \frac{\mathcal{E}}{R} = \frac{Bav}{R}\right)$

(2.) When the loop is moving through the region of homogeneous magnetic field at constant speed, no emf or current will be induced because the change of $\vec{B}$ in time $\left(\frac{\partial B}{\partial t}\right)$ is 0. Hence $\mathcal{E} = 0$ and $I_{ind} = 0$

(3.) When exiting the magnetic field, the magnitude of emf will be same as in (1.), meaning $\mathcal{E} = Bav$ (here in the derivation time $t = 0$ would be when the loop begins to exit). However, since the loop is exiting, the ~~time~~ mag. field is decreasing ⇒ change of $\vec{B}$ is in direction ⊙ ⇒ induced $\vec{B}$ will be opposite = ⊗ ⇒ inducent current will flow clockwise ↻ (I_3 in drawing)

$\boxed{2}$

[redacted]

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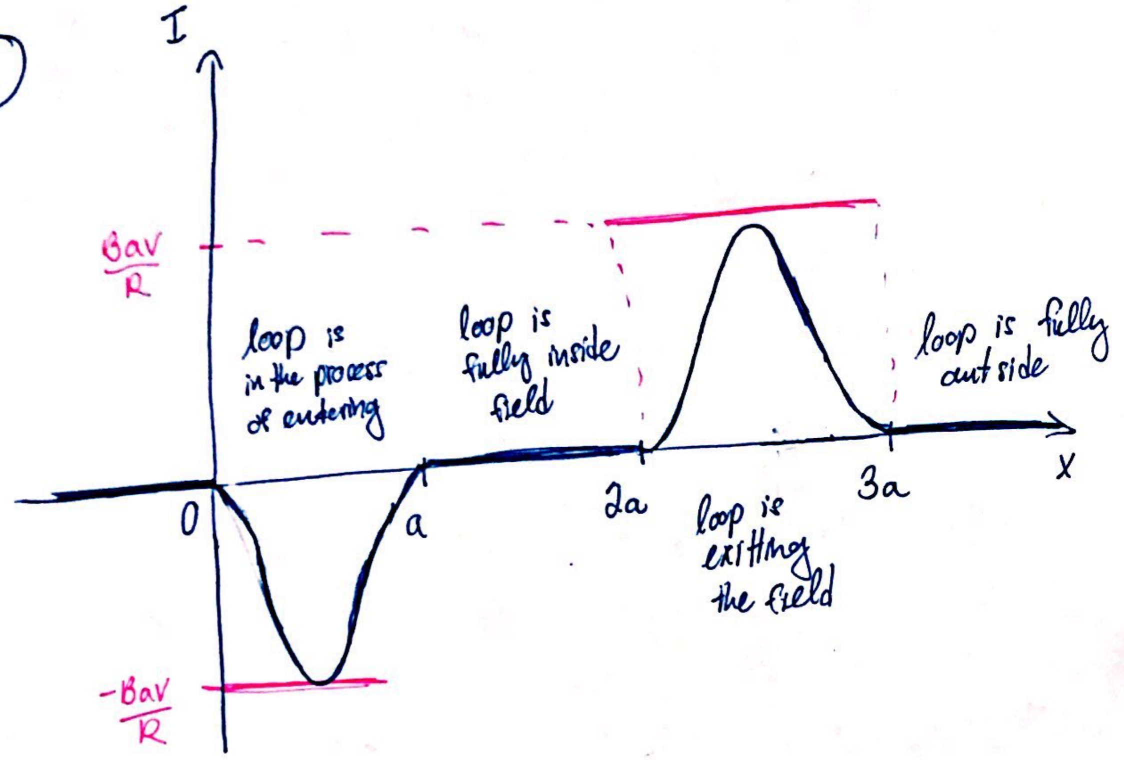
Q2 4.



Question 3 13/15

1. Coil under current produces magnetic field. When the current in the coil rapidly oscillates, the magnetic field will also be quickly changing ($\frac{\partial B}{\partial t} \neq 0$). According to Faraday's law, the change of magnetic field will create electric field and induce currents in the pot. These so-called Eddy currents produce heat (by law of conservation of energy) - work done by moving charges (current) against resistance is converted into ~~heat~~ heat by Joule's heating law. $P = VI = I^2R$

2. $\rho_{Cu} = 1.68 \times 10^{-8}\ \Omega m$
$\rho_{Al} = 2.65 \times 10^{-8}\ \Omega m$

The resistivity of aluminium is larger than the one of copper, so copper is a better conducting material, therefore it will be easier to produce Eddy currents in copper than in alumin. More currents will be induced in copper so although it will have smaller resistance, [larger current is more important ($P = I^2R$ $= \frac{V^2}{R}$)], copper is better material for heat production
(same emf induced)

3

[redacted]

3. $a >> b$

pan >> stove

↳R

$I_s(t) = I_0 \cos(2\pi\nu t)$ — [redacted]

$M = M_{sp} = M_{ps}$

By ~~...~~ the property of mutual induction, this is same as if I_s was running through I_p and we were inducing current in the small loop.

Magnetic field at a center of a wire loop of radius R:

$$\vec{B} = \frac{\mu_0 I}{2R}\hat{z} \quad \Rightarrow \quad \vec{B} = \frac{\mu_0 I}{2a}\hat{z}.$$

$$\Rightarrow \quad \phi = \int_S \vec{B}\cdot d\vec{a} \underset{?}{=} B\int_S da = \frac{\mu_0 I}{2a}\pi b^2$$

$\vec{B}$ is perpendicular to surface ($\parallel d\vec{a}$) and homogeneous

The flux is same in either symmetry case (induction $s \to p$ or $p \to s$)

So now when we know the flux, we can return to original problem.

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{\mu_0 I \pi b^2}{2a}\right) = -\frac{\mu_0 \pi b^2}{2a}\frac{d\left(I_0 \cos(2\pi\nu t)\right)}{dt} =$$

$$= +\frac{\mu_0 \pi b^2}{2a} I_0\, 2\pi\nu \sin(2\pi\nu t) = \frac{\mu_0 \pi^2 b^2 \nu I_0}{a}\sin(2\pi\nu t)$$

$$I_p = \frac{\mathcal{E}}{R} = \frac{\mu_0 \pi^2 b^2 \nu I_0}{aR}\sin(2\pi\nu t)$$

4. $\tau = ?$

$$P = VI = I_p^2 R = \cancel{R}\frac{\mu_0^2 \pi^4 b^4 \nu^2 I_0^2}{a^2 R^{\cancel{2}}}\sin^2(2\pi\nu t)$$

$$Q = W = \int_0^{\tau} P\, dt = \int_0^{\tau} \frac{\mu_0^2 \pi^4 b^4 \nu^2 I_0^2}{a^2 R}\sin^2(2\pi\nu t)\, dt = \frac{\mu_0^2 \pi^4 b^4 \nu^2 I_0^2}{a^2 R}\frac{\tau}{2}$$

$$\Rightarrow \tau = \frac{2Q a^2 R}{\mu_0^2 \pi^4 b^4 \nu^2 I_0^2}$$

[redacted]

[redacted]

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Q3

(5.) $\nu = 24\,kHz$ vs. $\nu = 50\,Hz$

From expression found in (4.), we can se[redacted] that the time it takes to deliver energy (heat) Q is inversely (quadratically) proportional to the frequency of ~~a~~ used ~~cu~~ alternating current.

This means that using higher frequency results in shorter ~~the~~ time of delivering same energy $\Rightarrow$ faster cooking ✓

Therefore using higher frequency can significantly lower the time it takes to deliver same energy.

Question 4

10/10

$$\vec{B}(s) = \frac{\mu_0 \varepsilon_0 s\, \omega V_0 \cos(\omega t)}{2d}\,\hat{\phi}$$

(1.) $\vec{J}_d = ?$

Ampere-Maxwell law: $\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J} + \vec{J}_d)$, since there's no current between the plates of the capacitor

$\Rightarrow \nabla \times \vec{B} = \mu_0 \vec{J}_d$

in cylindrical coordinates:

$$\nabla \times \vec{B} = \frac{1}{s}\frac{\partial}{\partial s}(s B_\phi)\,\hat{z} = \frac{1}{s}\,\frac{\mu_0 \varepsilon_0 \omega V_0 \cos(\omega t)}{2d}\,\frac{\partial s^2}{\partial s}\,\hat{z} =$$

$$= \frac{2s}{s}\,\frac{\mu_0 \varepsilon_0 \omega V_0 \cos(\omega t)}{2d}\,\hat{z} = \frac{\mu_0 \varepsilon_0 \omega V_0 \cos(\omega t)}{d}\,\hat{z} = \mu_0 \vec{J}_d$$

$$\Rightarrow \vec{J}_d = \frac{\varepsilon_0 \omega V_0 \cos(\omega t)}{d}\,\hat{z}$$

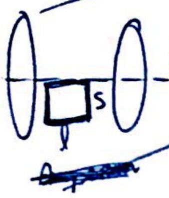
[redacted]

[redacted]

(2.) $\vec{E} = ?$

Faraday's law: $\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{a} = -\frac{\partial}{\partial t}$ [redacted] $\frac{\quad}{2d} \iint_0 s' \, ds\, dz$

in $\hat{\phi}$ dir. $\quad ds\,dz\,\hat{\phi}$



$$= + \frac{\mu_0 \varepsilon_0 \omega^2 V_0 \sin(\omega t)}{2d} \frac{s^2}{2} \cdot l = \oint \vec{E} \cdot d\vec{l} = \int_0^l E \, dz + 0 = E \cdot 2l$$

$$\Rightarrow \vec{E} = \frac{\mu_0 \varepsilon_0 \omega^2 V_0 \sin(\omega t) s^2}{8d}$$

(3.) For ~~...~~ there to be magnetic field between plates of a capacitor, it needs to charging/discharging ⇒ ~~the~~ constant ~~current~~ valtage would ~~do this only for very short time and then stop~~ not be creating any changes in the electric field between the plates, hence no magnetic field. ~~Also~~ More importantly, we are trying to measure the displacement current $\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$.
If there was constant valtange ⇒ const. $\vec{E}$ ⇒ no change in time ⇒ $\frac{\partial E}{\partial t} = 0$ ⇒ no displacement current is created.

(2.) $\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \Rightarrow \quad \vec{E} = \int \frac{\vec{J}_d}{\varepsilon_0} \, dt = \int \frac{\gamma_0 \omega V_0 \cos(\omega t)}{d \gamma_0} \, dt \, \hat{z}$

$$\vec{E} = \frac{V_0 \sin(\omega t)}{d} \hat{z}$$

[redacted]